POSITIVE EVOLUTION IN ECONOMIC FORECASTING. CASE STUDY: THE EVOLUTION OF A COMPANY'S CAPITAL

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Abstract

For the economic sciences, mainly for the planning and organization disciplines, the models display a range of varieties. For the last decades, though, the mathematics-based models seem to be the trend, partly because of their ability to rigorously concentrate the essentials and also to be programmed via the computer-based techniques, thus creating together an unheard of instrument of scientific investigation, a resourceful ‚extension' of human intelligence.

The article herein presents certain medium-term prognosis, via the dynamic modelling, elements of bifurcation theory and the Xpp software. The case study has been the object of a research contract with the business environment.

Key-words: *economic modelling, forecasting, dynamic analysis*

JEL Classification: C_{53}, C_{61},O_{21}

1. **Forecasting by dynamic models**

There is a similarity between the telescopes that widen our horizon without necessarily invalidating the previous discoveries in the close universe and mathematics that imparts new landscapes, while it is evolving from the current existence. Perspectives may change but not the truths.

The new theories discover the change reality once again. Time is no longer the reiteration of certain identical things, but becomes the carrier for differences. This means that the state of a system, at a certain point in time, is not included in its previous state, but there is a qualitative jump between two states.The sudden transitions, the ‚catastrophic' bifurcations do not occur in an uncertain manner. On the contrary, they derive from the conjunction of a multitude of factors that push the system in a specific direction and not in another one.

The dynamic models follow to highlight the temporal relations.The model operates with events and states that express the value of an attribute identifying the events occurence. The databases will help building diagrams of transition of the states that indicate all the operations that are specific to each type of object and relevant class.

As the research facilities fluctuate, the econometric studies have shifted from analysing the intervals to the dynamic models that may be studied via the latest theories.

The model, as an instrument of scientific knowledge, is used in multiple theoretical and practical disciplines. The knowledge derived from working with models and the attempt to implement them may unfold valuable concepts related to a certain issue and to the types of decisions that deem necessary. The simple recognition of the decision areas may be a major progress in many circumstances. Plus, using the models, there may come a recognition of the variables that may be controlled to influence the system performance, the relevant costs and their size, as well as the connection between costs and variables, including the options for important costs.

There is also a large number of factors that may affect the success opportunity within modelling. Some of them depend on the environment conditions, others on the modelling process management and still some more are associated with the nature of the model to be achieved.

The modelling of an economic process is a scentific means of finding the decisive factors that may intervene within the phenomenon. In order to set the importance level of such factors for the process, it is imperative to insert the most significant of them into the already built model in such a way to allow the mathematical conduct for them to be quantified, anytime possible.

Starting from the idea that any model is based on real data and parameters, it becomes crucial the fact to obtain trustworthy data that will allow a convenient representation of the reality via the model. Thus, the cyclical or periodical nature of the phenomenon is being studied, as case may be, along with the time horizon it is referring to.

The dynamic modelling is based on the fact that a system performing is represented by the knowledge of the interaction among the information flows, commands, human and material resources, etc. A dynamic model shows the behaviour of the complex systems, making apparent how their structure triggers the path and the behaviour in time.

The implementation of the dynamic models is mainly a simulation drill. Whether someone studies the cyclical behaviour, the alternative strategies, history interpretation, the evaluation of the model errors or anything else, the numerical simulation is the best instrument for them. For the small or the liniar systems, many questions may be directly answered to, sometimes including mathematical expressions; thus, for the general cases, mainly for the large size systems today, the numerical simulation is the only method to be used. Nevertheless, only one type of model simulation is considered as the ‚queen’ of them, i.e. forecasting (Popescu I., Ungureanu L., 2003).

2. **The presentation of the company**

The **Statia de utilaje pentru Constructii si Productie Industriala** – SA company has over 40 years of experience in concrete manufacturing, mortar, mineral units for pit ballast, metallic products of all kinds, furniture of double melaminated plywood and carpentry works and also in car rental – construction equipment. The company, due to the quality management policy, of the technical rigging and to the best quality products and services, has become one of the first suppliers for products and services on the civil constructions market, industrial and rod infrastructure in Craiova city and Dolj County.

Financial year	**Turnover** (RON)	Chart	**Profit** (RON)	Chart	**Employees**
2008	13 475 976		260 528		186
2007	10 825 579		309 358		166
2006	10 146 632		203 666		173
2005	6 708 331		163 344		166

3. **The extrapolation method**

Extrapolation is an explorative method and the most widely used in the quantitative forecasting. It consists in an inertial development of certain elements of the processes and phenomena where the future shows as an argumented extension of the present. The anterior evolution of a process – if there is an upholding of the conditions that have given a certain dynamics, conditioned by certain elements saving themselves or presenting future variations that are predictable – will equivocally trigger the future development of this process itself.

In other words, within this method, the future shows as an extension of the evolution monitored in the past. It is assumed that there will not be any fundamental mutations in the evolution of the phenomenon under study that would modify the structure of the precedent evolution. In this context, the use of extrapolation is recommended in the forecasting of the phenomena that keep their development pace and direction for a longer period of time. This method provides only an orientative image upon the future, a reason for which other methods should be used simultaneously.

The analytical extrapolation starts from a series of data, i.e. from a series of values in the dynamic series; these data are being processed during the dynamic series analysis and, based on this analysis, the future behaviour of the phenomenon is being estimated (the phenomenon described by the corresponding series).

In a nutshell, it is based on the models of prediction theory within the dynamic series (a chain of data that are derived from monitoring the quantitative features of a monitoring unit, during certain successive moments).

The extrapolation itself includes: the forecasting of the values in the parameters that interfere with the analytical expression of the curve describing the phenomenon – via economic and mathematical methods, as well as by reasoning or even analogies, the reporting of the present level of the phenomenon in the corresponding chart and the identification of the value present in the chart; the calculation of the values for the future (with some corrections), by using the curve in the process.

Thus, the method gives an overview upon the status of evolution of the phenomenon under study. This overview has to be, afterwards, well defined, corrected, supported with other methods. Then, a global analysis of the phenomenon will be carried out: after that, the laws governing the variable of the phenomenon will be set up and, thus, the essence of the phenomenon evolution will be grasped and correlations and hypotheses of the future evolution will be represented.

Together, we will go through the following stages: the **graphic representation of the series** and the calculation of the mathematical function that approximates the best the capital evolution.

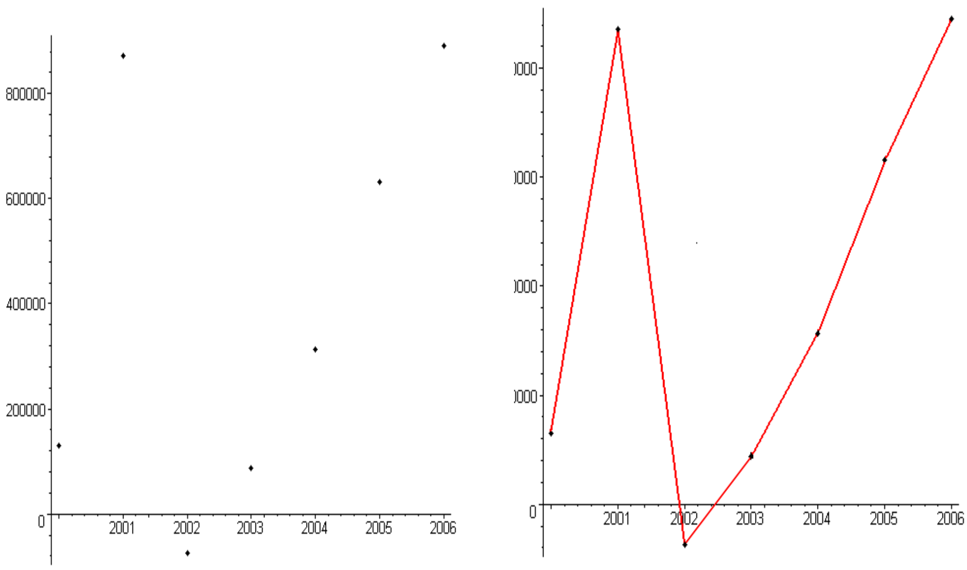


Fig. 1. *Graphic representation of the series*

According to the graphic representation, the evolution of this indicator is not well defined, thus we have to turn to calculation. It seems that the closest forms might be:

– a parabolic trend such as:

$$y = a + bt + ct^2$$

– a Tornqvist function-like trend:

$$y = bt\frac{t-c}{t+a}$$

We will be approaching each trend in itself and we will consider correct the only one where the sum of the deviation between the real values and the adjusted values is minimum:

a) the parabolic trend below:

$$y = a + bt + ct^2$$

The parameters *a*, *b* and c will be determined by using the smallest square method, where we obtain the following set of equations:

$$\begin{cases} na + b\sum t + c\sum t^2 = \sum y \\ a\sum t + b\sum t^2 + c\sum t^3 = \sum yt \\ a\sum t^2 + b\sum t^3 + c\sum t^4 = \sum yt^2 \end{cases}$$

The auxiliary calculation table will be:

Years	y	t	t^2	t^4	$t^2 y$
2004	872746	-2	4	16	3490984
2005	-75151	-1	1	1	-75151
2006	88192	0	0	0	0
2007	312507	1	1	1	312507
2008	630865	2	4	16	2523460
Total	1829159	0	10	34	6251800

In order to solve the system, we have the requirement $\sum t = 0$ and, then, the system becomes:

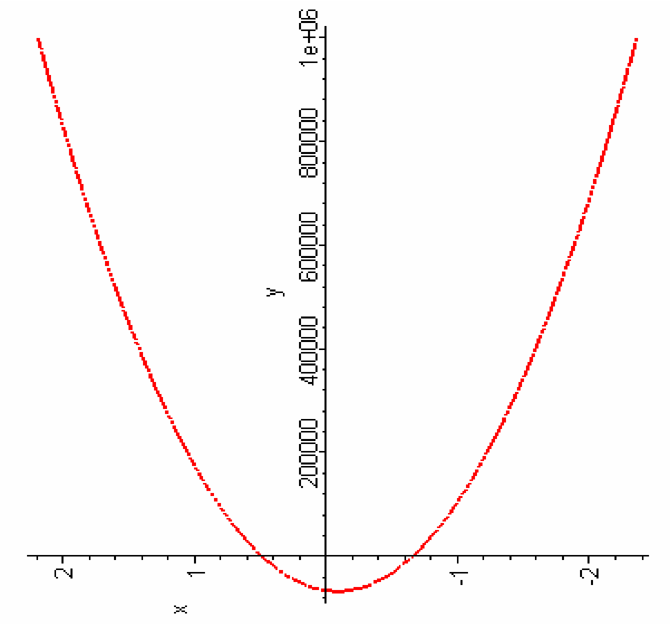
$$\begin{cases} na + c\sum t^2 = \sum y \\ b\sum t^2 = \sum yt \\ a\sum t^2 + c\sum t^4 = \sum yt^2 \end{cases}$$

The solution is:

$$a = -67614{,}408$$
$$b = 35565$$
$$c = 206413{,}857$$

The parabolic function reads like below:

y = 206413,857t^2+35565t-67614,408, and its graph is



b) Törnqvist function-like trend, in the form of:

$$y = bt\frac{t-c}{t+a}$$

The transform of this function is

$$y(t+a) = bt(t-c) \quad \Rightarrow \quad yt + ya = bt^2 - cbt \qquad cb = d$$

$$yt = bt^2 - ay - dt \,|: t \quad \Rightarrow \quad y = bt - a\frac{y}{t} - d$$

And the system of equations to determine *b*, *a* and *d*:

$$\begin{cases} \sum y = b\sum t - a\sum \frac{y}{t} - nd \\ \sum yt = b\sum t^2 - a\sum y \cdot d\sum t \\ \sum \frac{y^2}{t} = b\sum y - a\sum \frac{y^2}{t^2} - d\sum \frac{y}{t} \end{cases}$$

The results: *a*, *b* and *d*; $c = \frac{b}{d}$.

The calculation chart will be:

Years	y	t	t^2	y/t	y^2/t	y^2/t^2	y^2
2004	-0.075151	1	1	-0.075151	5647.672	5647.672	5647.672
2005	0.088192	2	4	0.044096	3888.914	1944.452	7777.828
2006	0.312507	3	9	0.104169	32553.541	10851.180	97660.625
2007	0.630865	4	16	0.157716	9949.766	24874.415	397990.65
2008	0.891392	5	25	0.178278	158915.94	31783.188	794579.69
Total	1.847805	0	10	34	300503.74	65334.843	

By replacing in the system and solving them, we have:

$$a = -0.46$$
$$b = 0.28$$
$$c = 0.64$$

The final function has the following form:

$$y = 0.28t\frac{t-0.64}{t-0.46}$$

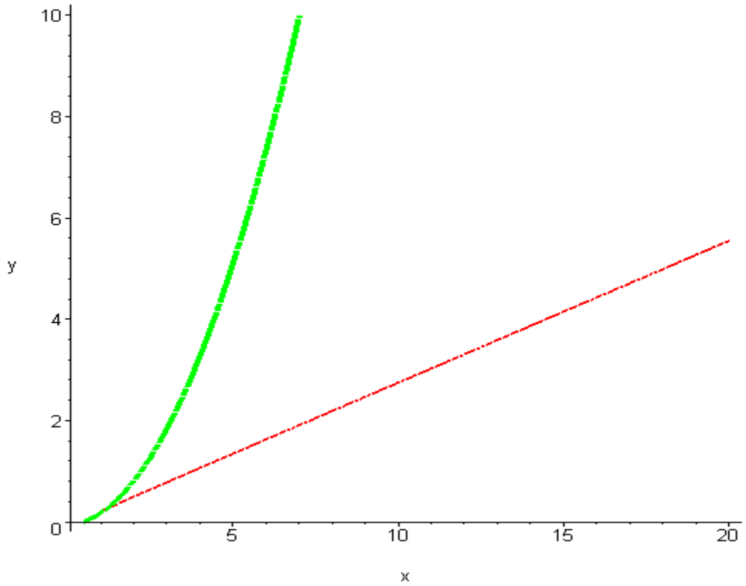
To select between the two variants of the best function, we will calculate the sum of the deviation in absolute value between the real and the adjusted values and will choose the function with a minumum value, i.e.:

$$\sum|y-Y| = \min$$

The deviations are calculated in the table below:

Years	Initial data	Törnqvist trend $\lvert y-Y_1\rvert$	Parabolic trend $\lvert y-Y_2\rvert$
1	-0.075151	0.01	0.01
2	0.088192	0.1	0.1
3	0.312507	0.23	0.53
4	0.630865	0.14	0.94
5	0.891392	0.45	1.36
Total		0.92	2.94

Since 0.92 <2.94, it means that the **Törnqvist function** is the most appropriate for adjusting the series.



The forecasting of the capital value, based on this function, is:

$$y_{2010} = 0.28 * 8\frac{8-0.64}{8-0.46} = 2.186\, mil\ lei$$

$$y_{2011} = 2.466\ mil\ lei$$

$$y_{2012} = 2.747\ mil\ lei$$, etc.

The mean square deviation is:

$$\sigma = \sqrt{\frac{(y_i - Y_1)^2}{5}} = \sqrt{\frac{0.01^2 + 0.1^2 + 0.23^2 + 0.14^2 + 0.45^2}{5}} =$$

$$= \sqrt{0.05702} = 0.238788 \text{ mil. lei}$$

and we may state the limits of the forecast values will be:

$$Y_L = \tilde{Y} \pm t\frac{\sigma}{\sqrt{n}}$$

4. **Dynamic modelling**

From an econometric perspective, the classical models, based on continuity, linearity and stability have proven inadequate to be able to represent economic phenomena and processes of a higher complexity. The researchers are compelled to dynamically follow-up these processes, to study the quantitative changes that occur between the involved economic values, as well as the results derived from them. Besides other features, the mathematical models allow the introduction of an isomorphism between the real and ideal economic system, represented by the model. Thus, it becomes possible to approach the unstable behaviour of the non-linear economic systems, underlining more often that the linearity and stability are actually particular cases of the economic evolution.

The modelling consists in building a representation of a variable fidelity degree for the real world or for a part of it. The understanding of the phenomenon or the reality segment under scrutiny, the deep knowledge as well as the action upon the phenomenon above motivate the rationing to turn to such representations.The most widely used language is the mathematical one.The use of the mathematical modelling helps to fundament a decision in efficiency conditions, giving the opportunity to think faster and better, without harming reality.

Starting from the idea that any model is based on real data and parameters, it becomes crucial the fact to obtain reliable data that will allow a convenient representation of the reality via the model.Thus, the cyclical or periodical nature of the phenomenon is being studied, as case may be, along with the time horizon it is referring to.

The non-liniarity of the evolution of a quite large number of phenomena in Physics, Biology, Ecology and Economy has led to shaping some modern and structured sciences, which are trying to approach, conceptualize and use another face of the reality, more fluctuant and dynamic. Such sciences are the result of integrating some models, theories and solving techniques of the non-linear, differential equations, of a change in perspective that generate new starting points for better understanding the phenomena herein. If the models are adequate, then the knowledge of their solutions will help us deduce the behaviour of the modelled phenomena. Even if every non-linear model has its own theory, they share some features, their strange behaviour of their solutions has a correspondent in the modelled phenomenon aspect. The fact that this behaviour has not been pointed out, it is because the complexity of the non-linear issued, which systematic study started only a few decades ago.

The dynamic modelling is based on the fact that the operation of a system is represented by knowledge in the interactions among the information flows, orders, human and material resources, etc. A dynamic model captures the behaviour of the complex systems, highlighting the way how their structure determines the trajectory and the behaviour in time.

The emergence of the non-linear dynamic theory has allowed the understanding and development of certain processes and methods that will bring us closer to the phenomenon in reality. The development of the singularity theory and the bifurcation theory has completed the multitude of means that we have in order to analyze and represent more and more complex dynamics, providing the possibility to examine certain systems that were difficult, if not impossible, to approach by traditional methods.The study of the non-linear dynamics is of a great interest, since the economic systems are non-liniar by excellence. Many of them include multiple discontinuities and an inherent instability, as they are constantly subject to external and internal shocks and perturbations.

For small values of parameters, visible changes of the variables may occur, i.e. bifurcations emerge that will steer the system to other trajectories.

The theory of bifurcation is a step forward, due to the fact that it owns a well-defined mathematical apparatus, which studies both the existence and the stability of the balance solutions, since an unstable balance one may not be noticed in reality.

Generally speaking, the models of economic growth explain the increase in products and income, thanks to the joint contribution of two production factors: capital (K) and labor (L). This increase dynamics or pace depends, on the one hand, on the capital and investments accumulation rate – hence, the K increase – and, on the other hand, by the technical progress (which allows to reduce the L percentage, as a result of the labor productivity increase). We will be exploring only one of the economic system models, its reply to the variations of the model data, of the parameters. By means of the data structures, diagrams of transitions will be drawn for the states that point out to all the operations specific to each type of object and relevant class.

4.1. *The mathematical model*

Among all the models that govern the evolution of the economic processes, we will take a look at one that consists in a Cauchy problem for a system of two ordinary first order differential equations in a real field. It describes the evolution of the capital in a company and of the workforce involved in it.

Let's say that K_t is the capital at the time t and L_t the workforce (number of the employed people). Then, the company turnover is y_t given by the production function $y_t = F(K_t, L_t)$.

The capital evolution is the function of the company development policy, by the income interests meant for investments, $(1-\delta_t)\pi_t$, where π_t is the net profit derived in the year t, a profit that may be entirely or partially distributed to the development, i.e. the amount left after the company stakeholders cover the dividends, in a percentage δ_t. Consequently, $\delta_t \pi_t$ is the dividends mass and $(1-\delta_t)\pi_t$ is the volume left for investments. By taking into account the capital depreciation with the average coefficient μ_t and by the income derived from the liquidation of the fading out assets to the fair currency value λ_t, the mathematical model of the company development thus results, where the basic equation of the capital evolution is below:

$$\dot{K}(t) = (1-\delta_t)\pi_t - \mu_t(1-\lambda_t)K_t$$

We have γ_t the growth rate of capital, expressed as a percentage. Since $\pi_t = \gamma_t y_t$, the result is

$$\dot{K}(t) = \gamma_t(1-\delta_t)F(K_t, L_t) - \mu_t(1-\lambda_t)K_t$$

We will also suppose that the variation of the workforce is

$$\dot{L}(t) = \alpha_1 K_t + \alpha_2 L_t - \alpha_0$$

and the company is characterized by a Cobb-Douglas production function type equation, $y_t = AK^{\alpha} L^{\beta}$. Should we consider that the production growth is stronger than the increase of the other factors, as a result of the efficiency increase (a characteristic fact for the more dynamic activity fields), i.e. a production with a growing physical outcome ($\alpha + \beta > 1$), we can consider the particular situation $y_t = AK^2 L$.

In this context, we will have the following system of equations:

$$\begin{cases} \dot{K} = A\gamma_t (1-\delta_t) K^2 L - \mu_t (1-\lambda_t) K, \\ \dot{L} = \alpha_1 K + \alpha_2 L - \alpha_0. \end{cases} \tag{1}$$

For this system, K and $L: \mathrm{R} \to \mathrm{R}$ are unknown functions that depend on the independent variable t (time).

We have the simplifying hypothesis regarding the consistency of the coefficients in (1) and obtain: $a = A\gamma_t (1-\delta_t)$, $b = -\mu_t (1-\lambda_t)$.

In light of the above, the evolution of the company's capital is governed by the Cauchy problem $K(0) = K_0$, $L(0) = L_0$ for the system:

$$\begin{cases} \dot{K} = aK^2 L + bK, \\ \dot{L} = \alpha_1 K + \alpha_2 L - \alpha_0. \end{cases} \tag{2}$$

While changing the variable $x = \beta_1 K,\ y = \beta_2 L$, where $\beta_1, \beta_2 \neq 0$ and $\beta_1 = \alpha_1 / \alpha_0$, $\beta_2 = 1/\alpha_0$, if $\alpha_0 \neq 0$ and $\alpha_1 \neq 0$, (2) thus becomes

$$\begin{cases} \dot{x} = cx^2 y + bx, \\ \dot{y} = x + \alpha_2 y - 1, \end{cases} \tag{3}$$

where $c = a\alpha_0^2 / \alpha_1$. In this context, the equations include only three parameters, *b*, *c* and α_2. This reduction will trigger, in the economic area, the highlight of certain expression, functions of primary economic parameters, which interfere with the capital and workforce evolution in a company. Thus, the same value of a new parameter corresponds to a large variety of value for the old economic parameters, which will obtain classes of equivalent economic situations.

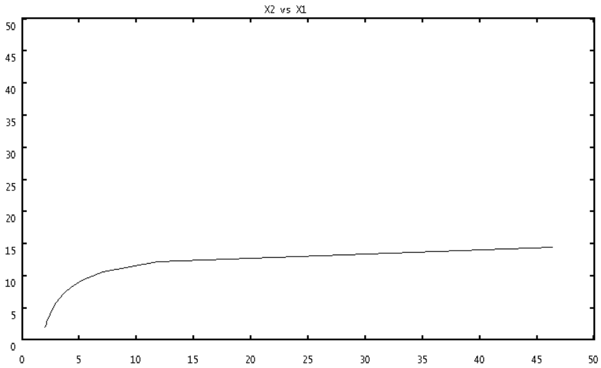
4.2. *Capital evolution*

This model shows a Cobb-Douglas type production function, with non-complementary elasticity coefficients, and a surging output of the population.

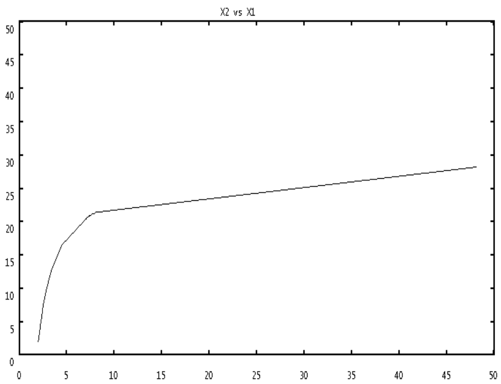
For the company under scrutiny, the capital depreciation does not occur, so the average coefficient is zero μ=0, the dividends coverage share of the company stockholders is δ=50%, the net profit percentage in the turnover is γ=4% and the income derived from the liquidation of the depreciated assets have the fair currency value λ=0.6 mil.lei. The initial data for the year of 2006 are $K_0 = 0.892, L_0 = 173, Y_0 = 1.36$.

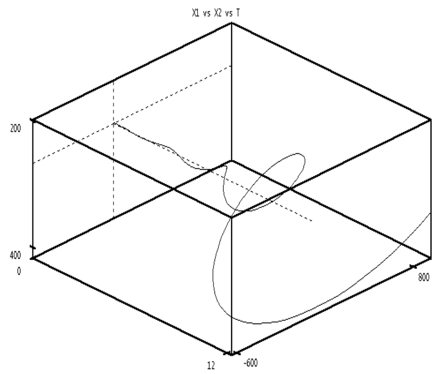
By replacing these numerical data in the model, we will have b=0. For α =0.1, we will have the parameter c variable and study the evolution of capital *K*.

1. c>0. Here, in this context of balance of the system, there is an unstable point, saddle type. Starting from the initial data, the beginning interval sees a slow increase, which reaches then 15 mil.lei.



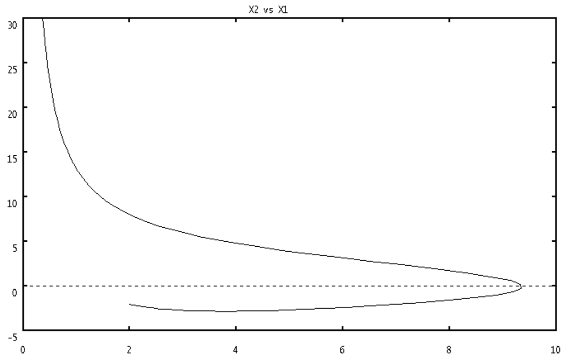
2. 0<c<-1/4. Here, the balance point is an attractive knot, where the capital has a calm state, and then it asymptotically goes towards 30 mil.lei.





3. c<-1/4. For this case, we notice a slow increase of the capital, then a short time of prosperity. Simultaneously, the workforce registers a very small

growth. A crisis is possible, which may lead to a capital decrease, to bankruptcy (negative values).



The next chart shows the diagram of the dynamic bifurcation related to the system in use.

5. **Conclusions**

In the long-term, the company has not registered the emergence of a crisis, of a steep decrease to lead to bankcrupcy or to massive layoffs of the workforce.

It is possible that, at one point in time, to have the economic activity stagnates and the production continuously slow down and unemployment rise. A reduction in the volume of the current production will take place.

The refreshing of the economic activity is closely linked to the renewal of the fixed capital, mainly its assets, leading to the exceeding of the determined minimum value. The revigoration of the economic activity in the investment process, in building new capacities and refreshing the current ones, fuels the demand for the production means.

As a conclusion, the company will be characterized by a general increase of the income. In such context, more and more favourable, business is getting better, with a perspective of consolidation. An investment process may be initiated, which will help update the current production abilities and create new ones. The process triggered by the surging demand for consumption goods is the decisive factor of production increase and of the employment percentage.

The economic forecasting has a theoretical and practical finality only by using modern, competitive methods of analysis and cuantification, able to catch the essence of the economic phenomena and processes being examined, to realistically evaluate their size, their future trends, as they are in an inter-conditioning relation with various factors.

Generally speaking, the more complex the activities are, the greater the need is for planning, search for strategies and formal and systematic actions. The economic field presents a high degree of uncertainty, where planning is essential for lowering it.

In a nutshell, the drafting of strategies in this field involves a clear and systematic structuring of the methods used to reach the desired objectives, via a correct distribution of the resources, long- or short-term. Any action in this area should take into account the most important aspects of planning, i.s. the familiarity with the product and with the consumer needs, as the 'decisional act represents a compromise between objectives and restrictions.'

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